Consider the sequence $3, -24, 192, -1536, 12288, \cdots$.

SCORE: _____ / 13 PTS

$\times(-8)$ $\times(-8)$ $\times(-8)$ $\times(-8)$ GEOMETRIC

[a] Find the formula for the n^{th} term of the sequence.

$a_n = 3(-8)^{n-1}$ 2

[b] Find the formula for the sum of the first n terms of the corresponding series $3 - 24 + 192 - 1536 + 12288 - \cdots$.

$S_n = \dfrac{3(1-(-8)^n)}{1-(-8)} = \frac{1}{3}(1-(-8)^n)$ 2

[c] Prove that your formula in [b] is correct **using mathematical induction**.

BASIS STEP: $S_1 = 3$

1 $\frac{1}{3}(1-(-8)^1) = \frac{1}{3}(9) = 3$ ✓

INDUCTIVE STEP: ASSUME $3 - 24 + 192 - \cdots + 3(-8)^{k-1} = \frac{1}{3}(1-(-8)^k)$, 1

1 FOR SOME PARTICULAR BUT ARBITRARY INTEGER $k \geq 1$

1 $3 - 24 + 192 - \cdots + 3(-8)^{k-1} + 3(-8)^k$

1 $= \frac{1}{3}(1-(-8)^k) + 3(-8)^k$

1 $= \frac{1}{3}(1-(-8)^k + 9(-8)^k)$

1 $= \frac{1}{3}(1 + 8(-8)^k)$

$= \frac{1}{3}(1-(-8)(-8)^k)$

$= \frac{1}{3}(1-(-8)^{k+1})$ 1

1 BY MI, $3 - 24 + 192 - \cdots + 3(-8)^{n-1} = \frac{1}{3}(1-(-8)^n)$ FOR ALL INTEGERS $n \geq 1$

FJ & GJ are friends, so when FJ opened a Twitter account, GJ started following it. The day the account was opened, **SCORE: _____ / 7 PTS** GJ received 11 tweets from FJ. Every day after that, GJ received 6 more tweets from FJ than he had received the previous day. One day, GJ noticed that, for the first time ever, he had received more than 200 tweets from FJ that day. As a result, GJ stopped following FJ's Twitter account, and recommended that FJ start attending Tweetheads Anonymous.

ARITHMETIC $d=6$

[a] How many days had FJ's Twitter account been open when GJ stopped following ?

$$ 2 \quad 11+6(n-1) > 200 \quad 1 $$
$$ 6(n-1) > 189 $$
$$ n-1 > 31\tfrac{1}{2} $$
$$ n > 32\tfrac{1}{2} $$
$$ n = 33 \quad 1 $$

GJ STOPPED FOLLOWING ON THE 33^{RD} DAY

[b] How many tweets had GJ received altogether by that time ?

$$ S_{33} = \frac{33}{2}\left(2(11)+6(33-1)\right) \quad 2 $$
$$ = 3531 \text{ TWEETS} \quad 1 $$

Expand and simplify $(2t^3-\sqrt{t})^4$. **<u>The coefficients in your final answer must be completely simplified.</u>** **SCORE: _____ / 5 PTS**

$$ (2t^3)^4 + 4(2t^3)^3(-\sqrt{t}) + 6(2t^3)^2(-\sqrt{t})^2 + 4(2t^3)(-\sqrt{t})^3 + (-\sqrt{t})^4 $$
$$ = 16t^{12} - 32t^9\sqrt{t} + 24t^7 - 8t^4\sqrt{t} + t^2 $$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$\frac{1}{2}$ POINT EACH

Find the coefficient of x^6 in the expansion of $(5x^2-3)^{43}$. **SCORE: _____ / 5 PTS**

You may write your final answer in factored form, as shown in lecture.

Your final answer must **<u>NOT</u>** contain ! or $C(n, r)$ (or equivalent) notation.

<u>NOTE: Do NOT use your calculator's ! nor $C(n, r)$ feature.</u>

$$ \binom{43}{r}(5x^2)^{43-r}(-3)^r = \binom{43}{r}5^{43-r}(-3)^r x^{2(43-r)} $$

$2(43-r)=6$

$r=40$

$$ = \binom{43}{40} 5^{43-40}(-3)^{40}x^6 \quad 1 $$
$$ = \frac{43!}{40!\,3!}5^3 3^{40} x^6 $$
$$ 1 \quad = \frac{43\cdot \overset{7}{\cancel{42}}\cdot 41\cdot \cancel{40!}}{\cancel{40!}\,3\cdot\cancel{2}\cdot 1}5^3 3^{40}x^6 = 43\cdot 7\cdot 41\cdot 5^3\cdot 3^{40}x^6 \quad 1 \quad 1 \quad 1 $$